PERFORMANCE IMPROVEMENTS FOR FINE-TUNED ADAPTIVE RECURSIVE FILTERS

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Abstract

Adaptive recursive filters are often implemented using direct-form realizations. However, direct-form realizations are known to have very poor sensitivity and roundoff noise properties, especially in the case of oversamplcd filters. Recently, a gradient-based technique was proposed for adapting a single column or row of a state-space filter where gradient signals are derived using only one extra titter. This paper presents results showing that these new structures are useful in oversampled filtering where an estimate of the final pole locations is known. It is shown, that these new adaptive structures can have much faster adaptation rates than direct-form realizations. As well, the adapted filters can have significantly better roundoff noise performance.

Introduction

There has been considerable interest in adapting recursive filters as is evident from the fact that numerous algorithms have been proposed for direct-form realizations [1-6] and lattice form [7,8] as well as biquad form [9]. In most of the adaptive recursive algorithms proposed to date, a single filter structure is intended to be used for all applications. However in some applications, one knows the approximate shape of the final transfer-function and the adaptive algorithm is only required to “fine-tune” the filter. One example of this type of application is in fixed-channel data equalization where the tuning of the adaptive filter need only account for manufacturing tolerances. The purpose of this paper is to show that significant advantages can be obtained in “fine-tuned” adaptive filter applications by using a recently proposed adaptive technique where only the coefficients in a single column or row of a state-space filter is adapted [10,11]. Specifically, it will be shown that in oversampled applications where final pole locations can be estimated, adaptive filters with much faster adaptation rates than those based on the direct-form structure can be obtained. As well, the noise performance of these new structures can be significantly better than the direct-form case. The advantage of this approach over using adaptive recursive lattice or biquad filters is the reduced amount of computation required to obtain the gradients.

Single Row or Column Adaptive Filters

An N’th order state-space digital filter can be described by the following equations:

$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b}u(n) \tag{1}$$

$$y(n) = \mathbf{c}^T\mathbf{x}(n) + du(n)$$

where $\mathbf{x}(n)$ is a vector of N states, $u(n)$ is the input, y(n) is the output and $\mathbf{A}, \mathbf{b}$, c and d are coefficients relating these variables. The matrix A is NxN, the vectors $\mathbf{b}$ and c are Nxl and d is a scalar. Using z-transforms, the transfer function from the filter input to the output is easily derived as

$$\frac{Y(z)}{U(z)} = \mathbf{c}^T(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{b} + d \tag{2}$$

From this equation, we see that the poles of the system are determined by the A matrix (the poles are simply the eigenvalues of **A**) whereas the zeros of the system are related to all four of the state-space matrices.

A block diagram of a state-space recursive adaptive filter is shown in figure 1 where the state-space coefficients now change with each timestep and hence are functions of the timestep “n”. The state-space system is shown as two separate blocks which correspond with the state-space describing equations. Specifically, the feedback matrix, **A,** and input summing vector, **b,** implement the first equation of a state-space system and create the state signals, x(n), as the outputs of the first block. These state signals together with the system input, u, are weighted using the output summing vector, c, and the output scalar, d, to obtain the filter output, y, at the output of the second block. The error signal, e(n), is the difference between the reference signal, $\delta(n)$, and the filter output, y(n). During adaptation, coefficients of the state-space filter are changed to minimize the mean squared value of the error signal. With the use of gradient signals, the LMS algorithm can be used to find a minimum of the mean-squared error performance surface [13].

It was shown in [10,11] that a LMS algorithm for adapting each of the state-space coefficients is realized by implementing the following equations.

$$A_{ij}(n+1) = A_{ij}(n) + 2\mu e(n)\alpha_{ij}(n) \tag{3}$$

$$c_i(n+1) = c_i(n) + 2\mu e(n)x_i(n) \tag{4}$$

$$d(n+1) = d(n) + 2\mu e(n)u(n) \tag{5}$$

where μ is a step size parameter which controls convergence of the algorithm and $\alpha_{ij}(n)$ are gradient signals which are simply the state signals of an extra gradient filter. Figure 2 shows how to obtain the gradient signals, $\alpha_{ij}(n)$, required to adapt the i'th column of **A. As shown in figure** 2, the gradient filter is simply the

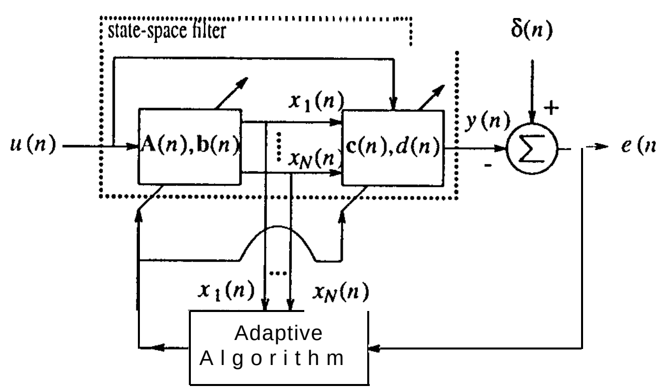


Figure 1: Adaptive state-space filter. The filter is shown in two separate blocks corresponding to the state-space describing equations.

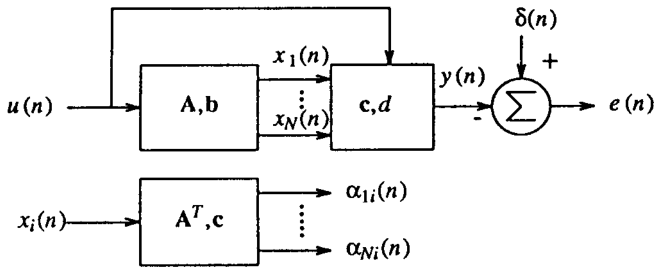


Figure 2: Generating the gradients for a single-column adaptive filter.

transposed system of the original state-space filter. To obtain a single row adaptive filter, two constraints must be satisfied. One constraint is that the input vector b must contain only one non-zero element and that the row of A to be adapted must correspond to the location of the non-zero b element. The second constraint is that the ***d*** element be zero. (One can relax this second constraint by using slightly incorrect gradient signals for small values of ***d***.) Figure 3 shows the method of obtaining the gradient signals for a single row adaptive filter where the input vector is shown as a basis vector, $\mathbf{v}_i$. It should be mentioned here that one should ensure that varying the chosen row row or column of A will result in enough degrees of freedom so that arbitrary pole locations can be obtained [10,11].

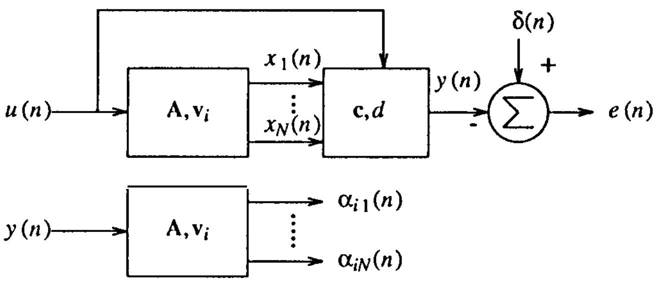


Figure 3: Generating the gradients for a single-row adaptive filter.

Choosing the Initial System to Adapt

There are two main advantages of using single-row or single-column adaptive filters over direct-form structures. Both of these advantages occur in oversampled applications where final pole locations can be estimated. One advantage is that the final adapted filter could have much better noise performance. This noise improvement can be obtained by choosing a much better structure than direct-form to implement the estimated pole locations. Then, if minor modifications are made to one column, or row, of the feedback matrix, the good noise properties of the structure should be maintained. The second advantage of using these new structures is a much improved convergence rate.

To illustrate the differences in expected convergence rates, we approximate the error performance surface by a quadratic function of the adaptive coefficients (as in [12]) in a close neighborhood of the operating point. The elements of the correlation matrix, R, corresponding to this approximate performance surface can be shown to be

$$R_{ij} = E[\alpha_i \alpha_j] \qquad (6)$$

where α_i is the gradient signal used to adapt the i'th feedback coefficient. This approximation indicates that one can estimate problems with the performance surface by estimating the degree of correlation between gradient signals.

Now consider the direct-form adaptive filter shown in figure 4 where the gradient signal used to adapt the a_i coefficient is shown as α_i [12]. Note that although this direct-form adaptive filter requires much less computation to obtain gradient signals than the method proposed in [5,6], the gradient signals are the same in both cases and thus the following reasoning holds for both types of direct-form adaptive filtering. From figure 4, it is not difficult to show that in the oversampled lowpass case, the gradient signals will be highly correlated assuming the input signal has a white noise characteristic. In fact, in the limiting case, as the system oversampling ratio goes to infinity, the gradient signals will be perfectly correlated.

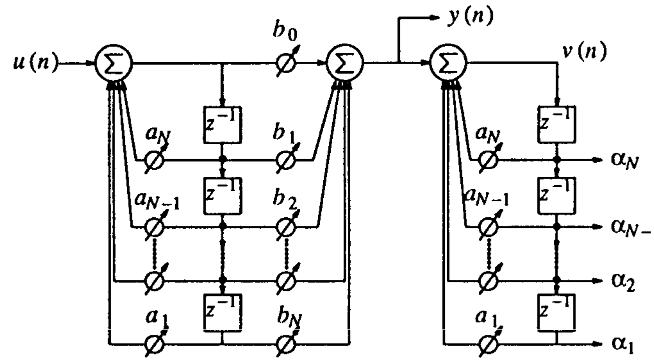


Figure 4: Direct-Form IIR filter and gradient signals.

Now, consider the single-row adaptive filter shown in figure 3 used in an oversampled lowpass application where estimates of the final pole locations are known. In this case, a state-space system can be designed having its poles equal to the estimated pole locations (only the A and $\mathbf{v}_i$ are required) such that the set of state outputs, x(n), are orthonormal for an input signal having a white noise characteristic [14]. As well, the set of states x(n) will have most of their power around the pole locations and thus will remain approximately orthonormal for an input signal that has a relatively flat spectrum response around the pole locations and low power elsewhere. This type of frequency characteristic is the spectrum one would expect for the output signal y(n) in this oversampled application with white noise at the input, u(n) leading to the conclusion that the gradient signals, $\alpha_i(n)$, will be approximately orthonormal. This approximately orthonormal set of gradients leads to much improved convergence rates as will be shown in some simulation results presented in the next section.

Finally, it should be mentioned that the design of orthononnal filters for arbitrary pole locations often results in dense matrices and therefore, this paper will make use of a variation of an ***orthonormal ladder filter*** structure presented in [15]. (For design details, see [11].) We shall refer to realizations obtained with this approach as "quasi-orthonormal filters" since the resulting realizations approach true orthononnal filters as the ratio of the sampling frequency to passband edge is increased. Of course, one is not restricted to using the quasi-orthonormal structure.

Simulation Results

We now present simulation results of adaptive filters with varying ratios of sampling frequency to passband edge frequency. The simulations are based on system identification applications where an N'th order adaptive filter is required to adjust its coefficients to match an N'th order reference transfer function. All the reference filters are derived from a third-order elliptic lowpass analog prototype with the following s-plane poles and zeros.

$$\textit{poles} = \{ -0.3226, -0.1343 \pm j0.91920 \} \qquad (7)$$

$$zeros = \{ \pm j\, 2.2105, \infty \}$$

The passband of the prototype has a 3 dB ripple with the passband edge normalized to 1 rad/sec.

To obtain oversampled digital filters with varying bandwidths, the bilinear transform [18] was applied to the analog prototype. The four resulting transfer-functions have ratios of sampling frequency to passband edge frequency of approximately 4, 8, 16, and 32. For each of these transfer-functions, three different structures for the adaptive filter are used: direct-form, single-row, and single-column adaptive filters. The single-row and single-column adaptive filters start from the quasi-orthonormal structure and then either the last row or column is adapted. The initial pole locations of the adaptive filters are three coincident poles on the real axis at a point chosen near the final pole locations. In all three cases, the initial pole locations are the same and the c vector and d scalar are both set to zero.

Table 1 lists the results of the different simulations. Note that as the reference filter becomes more oversampled, the direct-form structure takes much longer to adapt than either of the other two structures. As well, note that the noise measure, N_M, of the final adapted filter is higher for the direct-form case than the other structures in the cases of high sampling frequency to passband edge ratio. (The noise measure used is defined in [11] and is a slight variant of the measures presented in [17,18]). The graph in figure 5 summarizes the convergence times for the varying oversampled reference filters and different adaptive filter structures.

To better understand the reason why the direct-form filter adapts slowly at high oversampled rates, we look at a cross-section of the performance surface for the direct-form and single-row adaptive filters used to obtain the results in the last row of table 1. To obtain a three dimensional performance surface, we arbitrarily choose to vary only two coefficients; A_{31} and A_{32} while all the remaining coefficients are fixed to their optimum values. Figure 6 shows contour plots of the cross-section of the performance surfaces for both the aforementioned filters. It should be mentioned that the corresponding contour heights for both plots are the same but the axis scale for the direct-form filter is 10 times smaller than

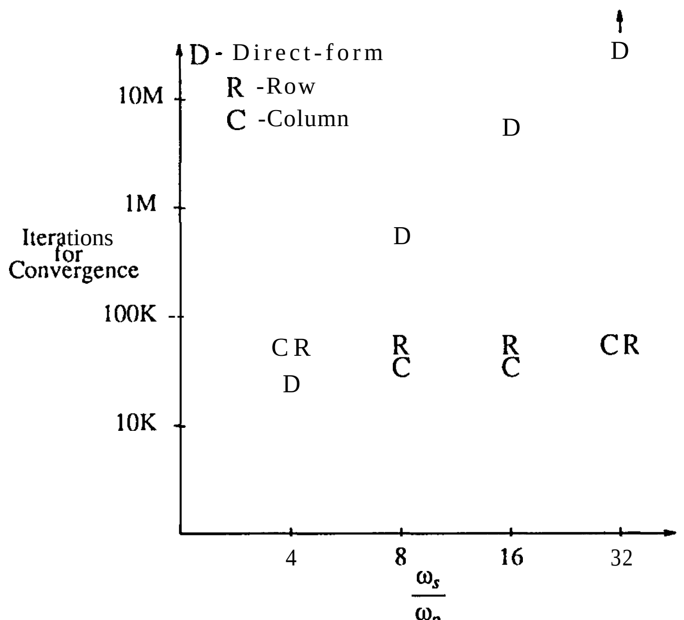
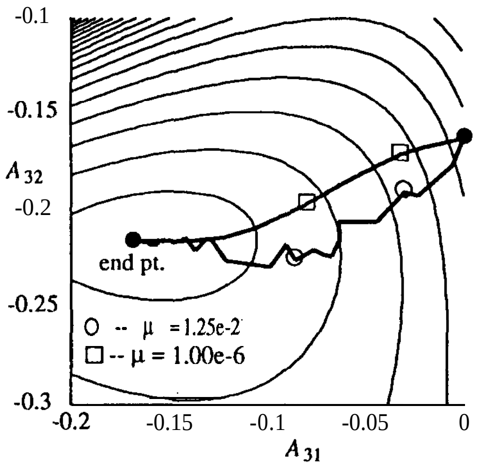
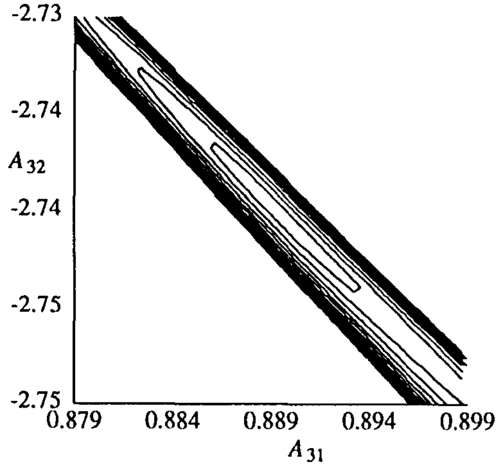


Figure 5: Convergence times of different structures for filters of varying bandwidths.



(b)



(b)

Figure 6: Examples of performance surface cross-sections for different filter structures.
(a) A single-row structure. Also shown are adaptation paths for a small (μ=1.00e−6) and large (μ=1.25e−2) step size. Note that for a small step size, the adaptation path follows the path of steepest descent, as expected.
(b) A direct-form structure (view expanded 10 times).

that for the single-row filter. This factor of 10 implies that the direct-form surface is actually much more ill-conditioned than that shown in figure 6. We can clearly see from figure 6 why the direct-form filter performs much worse than the single-row filter in this oversampled case. Also shown in figure 6(a) is the adaptation path for the single-row filter demonstrating that for small step sizes, the adaptation path does indeed follow the path of steepest descent. A larger step size adaptation path is also shown.

Conclusions

Single-column and single-row adaptive filter structures were shown have performance advantages over the direct-form structure in applications where final pole locations could be estimated. It is felt that these structure will be useful in applications where only "fine-tuning" of the adaptive filter is required. These adaptive filters are especially effective in the practical case of oversampled systems.

Table 1: Adaptation rates and noise measures for filters of varying bandwidths.

$\frac{\omega_s}{\omega_p}$	Transfer Function	Initial Poles		Direct Adapt	Row Adapt	Column Adapt
4	poles 0.5122 $0.06429 \pm j0.8625$ zeros $-0.6751 \pm j0.7377$	0	Step Size μ	0.01	0.0025	0.0028
		0	Iterations for Convergence	16K	60K	50K
		0	Noise Measure N_M	0.8	8 2	8 2
8	poles 0.7714 $0.6920 \pm j0.5904$ zeros -1.0 $0.09597 \pm j0.9954$	0.7	Step Size μ	2.8E-4	0.03	0.015
		0.7	Iterations for Convergence	500K	60K	35K
		0.7	Noise Measure N_M	4.8	2.4	5.0
16	poles 0.8788 $0.8872 \pm j0.3379$ zeros -1.0 $0.6581 \pm j0.7529$	0.8	Step Size μ	2.5E-5	0.01	0.0038
		0.8	Iterations for Convergence	6M	50K	30K
		0.8	Noise Measure N_M	49.	1.6	2.8
32	poles 0.9375 $0.9574 \pm j0.1775$ zeros -1.0 $0.9019 \pm j0.4318$	0.9	Step Size μ	1.0E-6	0.0125	0.0015
		0.9	Iterations for Convergence	>10M	40K	40K
		0.9	Noise Measure N_M	652.	1.4	2.6

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