DC Offsets in Analogue Adaptive IIR Filters

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Abstract

This paper presents approximate formulae relating coefficient error and excess mean squared error to DC offsets in analogue adaptive IIR filters where both poles and zeros are adapted. Simulation results are also presented verifying the formulae.

1. Introduction

Recently, experimental results for a discrete prototype analogue adaptive IIR filter were presented [l]. The discrete prototype consists of a state-space analogue filter where poles and zeros are adapted by adjusting state coefficients through the use of a gradient based LMS algorithm [2]. Although this analogue algorithm was presented for adaptive linear combiners [3], it has successfully been applied to an analogue adaptive IIR filter. The necessary gradient signals are obtained as the outputs of an extra filter which allows the adaptive algorithm to be implemented entirely with analogue components. However, as with any analogue system, DC offsets occur throughout the realization and therefore the effects of these offsets need to be addressed. Through experimentation, it was determined that some of the more critical locations for DC offsets to occur are in the coefficient update algorithm integrators. Towards understanding the effects of these integrator offsets, this paper presents approximate formulae relating coefficient error and excess mean squared error to DC offsets and a gradient correlation matrix.

The effect of DC offsets present in analogue adaptive linear combiners (where only zeros are adapted) has been previously investigated [4] and a formula derived giving the coefficient error due to DC offsets in coefficient update integrators. The main purpose of this paper is to show that this same DC offset formula can be used to give ***approximate*** results for adaptive IIR filters where both poles and zeros are adapted. Towards this end, a different approach is taken in deriving the DC offset formula in [4] so that an appropriate approximation during the derivation can be made.

2. Derivation of the DC Offset Formulae

Consider a general adaptive IIR filter shown in figure 1. The output of the programmable filter is subtracted from a reference signal, $\delta(n)$, to create an error signal, e(n). The adaptive

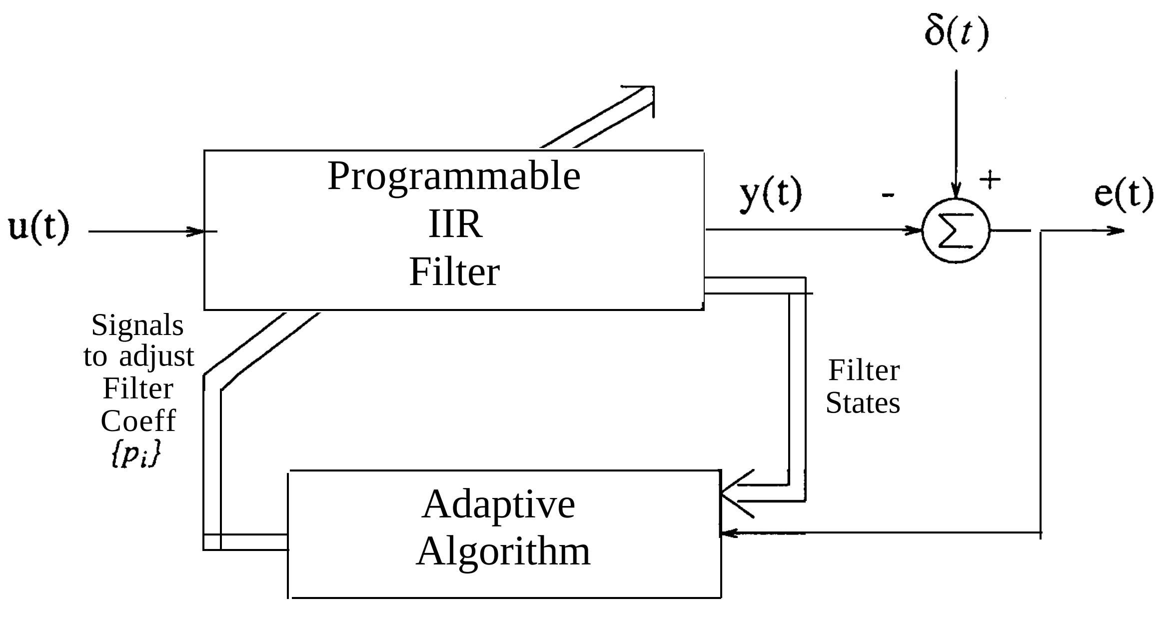

Figure 1: Adaptive IIR filter block diagram.

algorithm makes use of the error signal and filter states to adjust the programmable filter coefficients, $\{p_i\}$, in such a way as to minimize the mean squared value of the error signal. In other words, the purpose of the adaptive algorithm is to locate a minimum in an error performance surface by adjusting the filter coefficients. The error performance surface is defined to be the surface which is created by measuring the mean squared value of the error signal as the filter coefficients are varied.

One approach to finding a minimum in an error performance surface is to use the method of steepest descent. Unfortunately, with the standard steepest descent method, one requires a partial derivative of the mean squared error with respect to the filter coefficients. To circumvent this problem, the least-mean-squared (LMS) algorithm was developed [5] where the instantaneous error squared signal is used to approximate the mean squared error. With this approach, the following LMS update formula for the coefficient p_i is obtained [3]

$$p_i(t) = 2\mu\int_0^t \left[e(\tau)\frac{\partial y(\tau)}{\partial p_i} \right] d\tau$$

where μ is a small positive parameter which controls the rate of convergence.

Unfortunately, the above update formula can not be realized exactly and DC offset terms will always be present in analogue implementations. With a DC offset term present, the i'th coefficient update formula becomes

$$p_i(t) = 2\mu\int_0^t \left[e(\tau)\frac{\partial y(\tau)}{\partial p_i} + m_i \right] d\tau$$

where m_i is the DC offset for the i'th update formula. A block diagram for this coefficient update formula is shown in figure 2 where the DC offset is injected as a separate signal so that the integrator may be considered ideal. Note that with such a conceptualization, the DC offset term need not come from only the offset of the integrator. If both the error signal and gradient

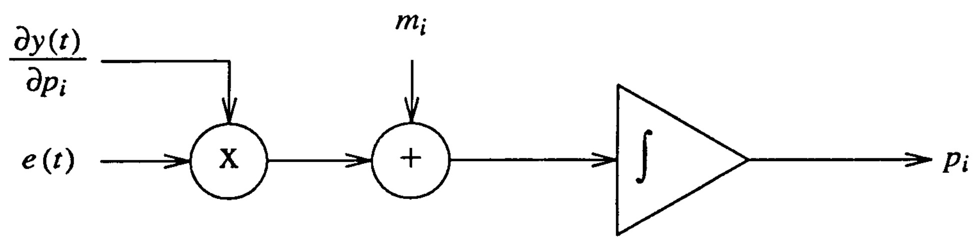

Figure 2: Block diagram of coefficient update formula with DC offset, m_i.

signal have offsets, then these DC signals correlate with each other resulting in an additional DC offset. By defining m_i to also include this offset signal, we may consider both the error and gradient signals to be ideal with respect to DC offsets.

When an adaptive filter is at steady state, the expected value of the coefficient signal p_i is a constant value implying that the expected value of the signal into the integrator must be zero. Thus at steady state, the following equation holds

$$E\left[e(t)\frac{\partial y(t)}{\partial p_i} + m_i\right] = 0$$

where $E[\bullet]$ denotes expectation. Since the expected value of a DC signal is the DC level, we can write

$$E\left[e(t)\frac{\partial y(t)}{\partial p_i}\right] = -m_i \tag{1}$$

We now make use of the fact that with the LMS algorithm, the inside of the expectation operator in equation (1) is the instantaneous estimate of the derivative of the mean squared error with respect to the parameter p_i, or in mathematical terms,

$$e(t)\,\frac{\partial y\,(t)}{\partial p_i} = \frac{-1}{2}\,\frac{\partial e^2(t)}{\partial p_i} \tag{2}$$

Substituting equation (2) in equation (1) above and swapping the expectation and derivative operators, we also have the following condition at steady state.

$$\frac{\partial E\,[e^2(t)]}{\partial p_i} = 2m_i$$

This formula implies that when no DC offset is present in the i'th update formula $(m_i = 0)$, the adaptive filter settles at a point where the partial derivative of the performance surface with respect to the i'th coefficient is zero. This is precisely the condition for finding a minimum. However, in the case of a non-zero DC offset, the adaptive filter settles at a point where the same partial derivative is at a value equal to twice the DC offset. In other words, the filter settles at a position where the error is slightly correlated with the gradient signal in order to cancel the effect of the DC offset, as seen from equation (1). Note that a DC offset forces the filter coefficients to be incorrect which implies an error in the programmable filter's transfer function at all frequencies (not just at DC).

For the adaptive IIR filter shown in figure 1, consider the case where there are N filter coefficients, $\{p_i\}$. Now making the assumption that only small coefficient changes occur due to DC offsets, at steady state the error signal can be written as

$$e\,(t) = \delta(t) - y^*(t) - \sum_{i=1}^{N} \frac{\partial y\,(t)}{\partial p_i}\Delta p_i \tag{3}$$

where $y^*(t)$ is defined as the optimum output which causes the minimum mean squared error and Δp_i is defined to be the change due to DC offsets in coefficients from their optimum values. Making the use of vector notation, we can write the gradient signals as a vector, x(f), where

$$x_i(t) \equiv \frac{\partial v(t)}{\partial p_i} \quad for\ i = 1 \text{ to } N$$

and the change in coefficients as the vector $\mathbf{q}$ where

$$q_i = \Delta p_i$$

With this notation, we can write equation (3) as

$$e(t) = \delta(t) - \mathbf{y}^*(t) - \mathbf{x}^T(t)\mathbf{q}$$

Since we are interested in finding the excess mean squared error and coefficient errors that result from DC offsets, without loss of generality, we make the assumption that the mean squared error equals zero if all the DC offsets are zero. Making this assumption implies that the optimum filter output, y*(r), equals the reference signal, $\delta(t)$ and therefore the excess error signal can be reduced to simply

$$e(t) = -\mathbf{x}^T(t)\mathbf{q} \tag{4}$$

Now writing the DC offsets as a vector **m, we** can apply equation (1) above for the set of DC offsets to obtain

$$E[\mathbf{x}(n)e(n)] = -\mathrm{m} \tag{5}$$

Combining equations (4) and (5) results in

$$E[\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{q}] = \mathbf{m} \tag{6}$$

Assuming the gradient signals, $\mathbf{x}(t)$, form an independent set, one can always find an orthonormal set of signals, v(r), related to $\mathbf{x}(t)$ using a matrix S such that

$$\mathbf{x}(t) = \mathbf{S}\mathbf{v}(t) \tag{7}$$

Substituting equation (7) in equation (6), we obtain

$$\mathbf{S}\mathbf{S}^T\mathbf{q} = \mathbf{m} \tag{8}$$

where we have made use of the fact that the set of signals in $\mathbf{v}(t)$ form an orthonormal set and therefore

$$E[\mathbf{v}(t)\mathbf{v}^T(t)] = \mathbf{I}$$

where **I** is the identity matrix.

We now define a gradient correlation matrix, R, as

$$\mathrm{R} \equiv E[\mathbf{x}(t)\mathbf{x}^T(t)]$$

With this definition of the correlation matrix, R, one can find the following relationship between R and S.

$$\mathbf{R} = \mathbf{S}\mathbf{S}^T$$

Therefore equation (8) can be written as

$$\mathbf{q} = \mathbf{R}^{-1}\mathbf{m} \tag{9}$$

Thus, with the use of the gradient correlation matrix, R, this equation gives the error in coefficient values due to DC offsets. Note that this is the same equation derived for the adaptive linear combiner case [4] except that this formula is exact in the linear combiner case whereas, here, it is only an approximation due to the assumption made for the error signal formula.

To obtain the excess mean squared error, $||e||^2$, due to DC offsets, we use the definition for the mean squared error and perform the following manipulations,

$$\begin{aligned} ||e||^2 &= E[e(n)e(n)] \\ &= E[\mathbf{q}^T\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{q}] \\ &= E[\mathbf{m}^T\mathbf{R}^{-T}\mathbf{S}\mathbf{v}(n)\mathbf{v}^T(n)\mathbf{S}^T\mathbf{R}^{-1}\mathrm{m}] \\ &= \mathbf{m}^T\mathbf{R}^{-1}\mathbf{m} \end{aligned} \tag{10}$$

Note that in the case where all the gradient signals are orthonormal (ie. R equals the identity matrix), the excess mean squared error is simply the sum of the squares of the DC offsets. Also note that the level of the input signal affects the excess error through the correlation matrix R.

3. Simulation Examples

As a test for this error formula, some simulations of adaptive ***digital*** filters were performed. Digital filters were are used in the simulations since they are much easier to simulate than analogue equivalents. In the first example, a third order adaptive linear combiner with DC offsets was simulated where the reference transfer function was $z^{-2} + z^{-1} + 1$. The input correlation matrix R for the simulation was chosen to be

$$\mathbf{R} = \begin{bmatrix} 1 & 0.8 & 0.9 \\ 0.8 & 1 & 0.72 \\ 0.9 & 0.72 & 1 \end{bmatrix}$$

and the DC offset vector **m** was arbitrarily set to

$$\mathbf{m} = \begin{bmatrix} 0.0\text{[illegible]} \\ 0.0\text{[illegible]} \\ -0.03 \end{bmatrix}$$

For this example, the calculated excess root mean squared error (RMSE) using equation (10) was 0.0922 and the simulated RMSE was 0.092. This is certainly a close agreement as one expects for the adaptive linear combiner case.

To check the validity of this formula for the IIR case, a second order example with DC offsets present was simulated. In this simulation, only the bottom row of the A matrix was adapted while all other parameters remained equal to the optimum values. The reference filter had the state-space describing equations:

$$A = \begin{bmatrix} 0 & 1 \\ -0.6 & 1.2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0.41 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad d = 0$$

Defining x(n) to be the vector of gradient signals required to adapt the bottom row of A, the correlation matrix, R, was found to be

$$\mathbf{R} = \begin{bmatrix} 0.998 & 0.8\ 16 \\ 0.816 & 0.998 \end{bmatrix}$$

Note that this correlation matrix shows high correlation between gradients and therefore a large curvature in the performance surface. With offsets equal to 0.01 and -0.01 for m_1 and m_2, respectively, the simulated and calculated RMS errors were 0.036 and 0.0332, respectively. The bottom row of the **A** matrix settled at the coefficient values -0.575 and 1.175. This example shows a reasonable agreement between the calculated and simulated values. However, by decreasing the offset, the accuracy of the small change approximation in equation (10) is improved and therefore an even closer agreement should be obtained. Decreasing the offsets to 0.001 and -0.001, the simulated and calculated RMS errors were 0.0033 and 0.00332, respectively, which is certainly a close agreement. For this simulation, the bottom row of the **A** matrix settled at the coefficient values -0.598 and 1.198.

Finally, note from equation (10) that the value of the excess error due to DC offsets is proportional to the inverse of the correlation matrix, R. This fact implies that the excess error will increase as the states become more correlated since the matrix R will become more ill-conditioned. This increased excess error is one reason to look for adaptive IIR structures with orthonormal gradients for analogue realizations.

4. Conclusions

This paper presented approximate formulae relating coefficient errors and excess mean squared error to DC offsets in analogue adaptive IlR filters. Simulation results were also presented verifying the derived formulae.

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